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The biography of Euler and Gauss as inspiration to study science

La biografía de Euler y Gauss como inspiración para estudiar ciencias



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Abstract

There is a latent educational problem, mainly in the field of exact sciences, where there are indicators of high failure rates and poor student performance; Likewise, there is a lack of interest in fundamental issues. Perhaps one of the main causes of this is that the teaching-learning methods have not been updated, that is, we have not incorporated new technologies. In addition to the above, we consider that there is another element that consists of seeking the motivation of the student body through knowing the biographical history of the great scientific figures. The aim of this work is to propose the inclusion of a series of topics from two of the great mathematicians in history: Leonardo Euler and Carl Friedrich Gauss, so that they can be integrated into the courses, seeking to increase the interest of the boys. Therefore, a set of lessons is suggested for Mathematics and Physics courses at the preparatory level. The following topics can be explained from Euler: Euler's straight line, Prime numbers, Formula of polyhedral, Euler identity and convergent series. From Gauss: sum of consecutive numbers, formula of prime numbers, normal distribution function, Gauss surfaces, method of least squares, solution of systems with Gauss Jordan elimination, and finally, some generalizations of his contributions in electromagnetism and astronomy.

Keywords: Polyhedra, Prime Numbers, Exponential Function, Logarithm Function, Normal Distribution, Systems of Linear Equations, least squares

Resumen

Hay un problema educativo latente, principalmente en el campo de las ciencias exactas, donde se tienen indicadores de alto índice de reprobación y de un pobre desempeño de los alumnos; de igual manera, hay falta de interés en temas fundamentales. Quizá, una de las principales causas de esto, es que no se han actualizado los métodos de enseñanza-aprendizaje, es decir no hemos incorporado nuevas tecnologías. Aunado a lo anterior, nosotros consideramos que hay otro elemento que consiste en buscar la motivación del estudiantado a través de conocer la historia biográfica de los grandes personajes científicos. El objetivo del presente trabajo, es proponer la inclusión de una serie de temas de dos de los grandes matemáticos de la historia: Leonardo Euler y Carl Friedrich Gauss, para que se integren a los cursos, buscando aumentar el interés de los muchachos. Por tanto, se sugiere un conjunto de lecciones para los cursos de Matemáticas y Física a nivel preparatoria. De Euler se pueden explicar los siguientes temas: Línea de Euler, Números primos, Formula de poliedros, identidad de Euler y serie convergente. De Gauss: suma de números consecutivos, formula de números primos, función de distribución normal, superficies de Gauss, método de mínimos cuadrados, solución de sistemas con la eliminación de Gauss Jordan, y por último, algunas generalizaciones de sus contribuciones en electromagnetismo y astronomía.

Palabras clave: Poliedros, Números Primos, Función Exponencial, Función Logaritmo, Distribución Normal, Sistemas de Ecuaciones Lineales, mínimos cuadrados

1. Introduction

In the middle or high school educational level, the study, and, therefore, the earliest understanding of the world of science begins. In that context, a student may eventually have the good fortune that his teacher has the virtue of narrating the biographies of the great masters, for example, Newton, Euler, Gauss, and Einstein, who are recognized among the most popular key figures of scientific development. There are countless legends and well-remembered facts about these masters. Among many anecdotes, the image of Gauss, who responds to the questioning of his elementary school teacher on how to obtain the sum of a set of consecutive numbers, or the apple falling just in front of Newton, revealing the mysteries of gravity, stand out.

To promote the interest and enjoyment of science among children and young adolescents, it is essential to make changes in the teaching of science, to cultivate a more accessible and attractive didactic approach. In this respect, a group of professors at the University of Sonora, Mexico, have proposed the inclusion of lessons at the pre-university level that have as their main distinctive feature the highlighting of the achievements of Euler, Gauss, Newton and other distinguished figures. This paper discusses how those materials may be.

The idea is to stimulate appreciation for science from an early age. However, there is an obstacle to achieving that purpose, which is the widespread feeling among students that it is an intimidating challenge to study exact sciences, so the proposals should try to end that status quo. One exercise that could make these subjects more joyful and fun is to present the contributions of the geniuses in an accessible way. The aim is to try to inspire children to explore and enjoy scientific knowledge.

This essay includes some contributions of Euler and Gauss. It is proposed to explain in the simplest way Euler's straight line and the formula of polyhedral. Gauss presents the sum of consecutive numbers, the bell of Gauss, also in an elementary way, the units to measure the strength of magnets in the study of magnetism are included, also a little of hyperbolic geometry is mentioned, and a brief biographical sketch of each is presented.

Leonhard Euler, a Swiss genius of the 18th century (century of enlightenment), who did much of his work in St. Petersburg, Russia. He has left an immeasurable mathematical legacy. His contribution is so profound, that his name is always considered among the greatest mathematicians in history. One of his many achievements is in the field of arithmetic and number theory. Euler made fundamental contributions to the study of prime numbers and his concepts are essential in cryptography, which is used in the development of algorithms in computer security. Another popular contribution is Euler's famous formula, which shows that, for any regular polyhedron, the number of vertices minus the number of edges plus the number of faces is always equal to 2. Experts say that Euler established the way mathematics is understood and applied in the contemporary world, and his legacy continues to inspire scientists.

Among the books about him that can be found, there is: *Leonhard Euler: Mathematical Genius in the Enlightenment* (Calinger, 2019); it offers a detailed biography of Euler, exploring his life and contributions. Another book is: *Euler: The Master of Us All* (Dunham, 2006), which presents Euler's life in an accessible and entertaining way. In Mexico, the excellent book *Legado matemático de Leonhard Euler* is available and recommended for further study (Anzaldo Meneses et al., 2007).

Carl Friedrich Gauss, born in 1777 in Germany, is considered one of the most brilliant minds in the history of science. His studies were in a wide range of disciplines, from number theory to astronomy. There is a consensus among experts that his genius has influenced the understanding and advancement of scientific knowledge. Like Euler, Gauss was key in the development of arithmetic, as he unlocked for mankind many secrets of number theory. Gauss also left his mark in the field of geometry, where his exploration of the foundations of geometry contributed to the evolution of the understanding of space and geometry in modern mathematics (Perero, 1994).

He also had a great influence on astronomy, as his work is widely recognized as a jewel in the understanding of the solar system. Gauss applied mathematical methods to analyze astronomical observations and predict the orbits of planets and asteroids (Perero, 1994). The *prince of mathematics* developed a big amount of theory, but he also has a characteristic that few scientists possess, i.e., linking his discoveries to the world of work and reality. His work has a great impact on the universe of applications of scientific knowledge. For example, the famous Gauss curve in statistics.

One of the books is *Carl Friedrich Gauss: Titan of Science* (Dunnington, 2012), a biography that provides a detailed overview of Gauss' life and work and contains explanations of his contributions to mathematics and physics. Another is: *Gauss: A Biographical Study* (Bühler, 1981), which provides a more general look at Gauss's life and discusses his impact on the history of mathematics.

The following is a brief explanation of topics that could be explained to students in the middle or high school level. From Euler, the Euler's straight line and the formula for polyhedral are presented; from Gauss, the sum of consecutive numbers, the Gauss curve in statistics and the Gaussian measure of magnets are explained.

2. Contributions

2.1. Some contributions of Leonard Euler

Euler's straight line is a geometric concept that is related to the properties of a triangle (Dunham, 2006). It was discovered by Euler and refers to a line that passes through the so-called associated notable points of a triangle. To understand the Euler's straight line, first, we must remember the remarkable points of a triangle (Figure 1):

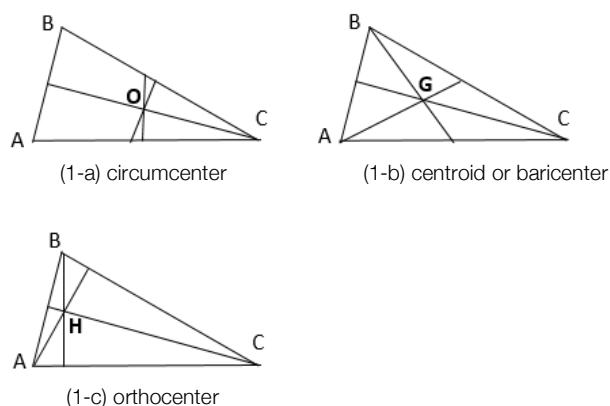
Barycenter (G): It is the point of intersection of the medians of the triangle. The median is a line that connects a vertex of the triangle with the midpoint of the opposite side. The centroid of a triangle (or center of mass) G is the point where the three medians of triangle intersect. The medians (m_a , m_b and m_c) are segments that connect one its vertices with the center of the opposite side.

Circumcenter (O): It is the center of the circumcircle circumscribed to the triangle. The circumscribed circle is the one that passes through the three vertices of the triangle.

Orthocenter (H): It is the point of intersection of the heights of the triangle. The height is a line perpendicular to one side of the triangle passing through the opposite vertex.

Figure 1

Euler's straight line

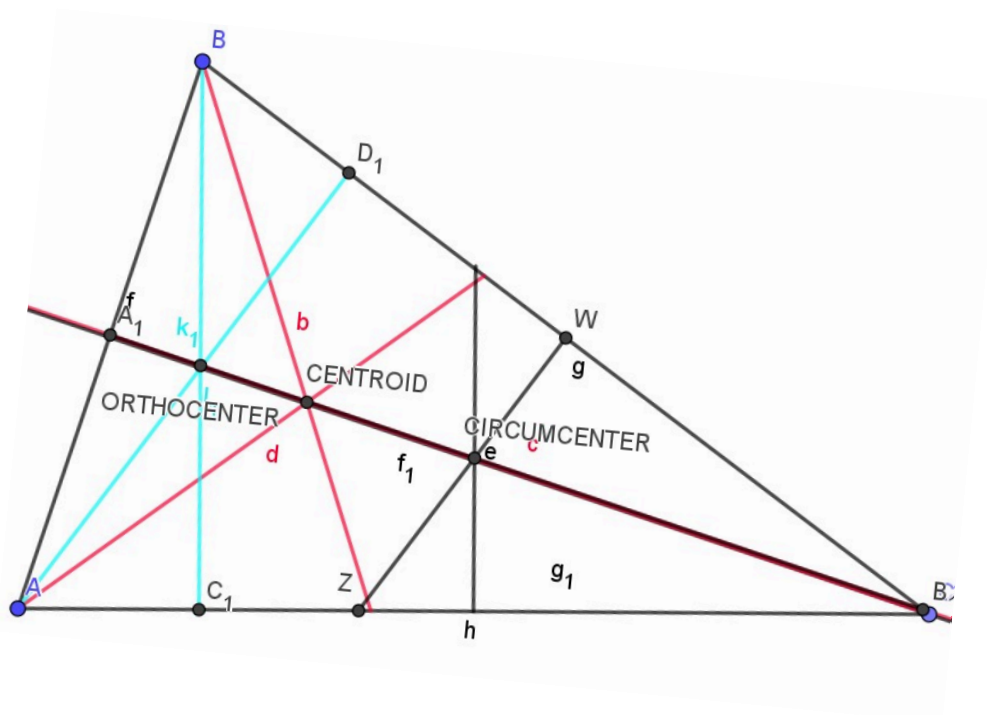


Note. Adapted from Universo Fórmulas (2024). The three similar triangles are shown to observe the three points “OGH” of the Euler line: A didactic way of presenting this topic.

What Euler discovered was that these three remarkable points (G, O, and H) are aligned on a straight line, which is known as the Euler's Straight Line. It is important to note that the famous line always passes through these three remarkable points, regardless of the type of triangle, whether it is considered equilateral, isosceles, or scalene (Dunham, 2006) (Figure 2).

Figure 2

The points that form Euler's straight line "HGO" in a triangle



2.2. Euler's Theorem for polyhedral

Is a precious concept in geometry. This theorem, attributed to Euler, establishes a relationship between the vertices (V), edges (E) and faces (F) of a convex polyhedron (Figure 3, Figure 4 and Table 1). The formulation of Euler's theorem for polyhedral is expressed by the following equation: $V - E + F = 2$.

Figure 3

Shape of the regular Hexahedron (cube), and its importance of its geometric characteristics

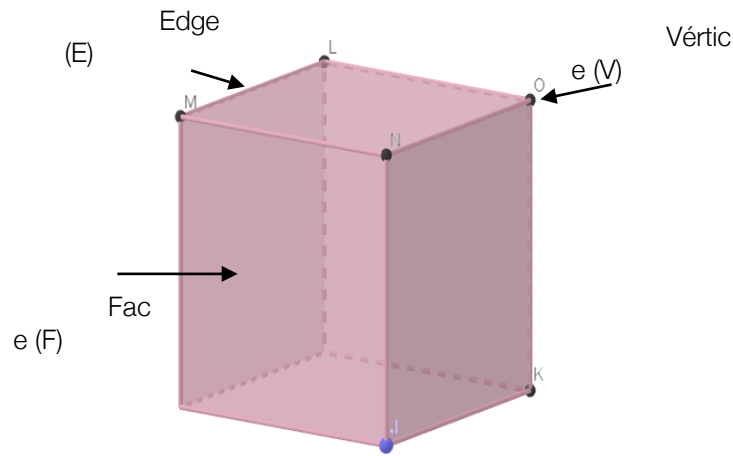


Figure 4

Regular polyhedrons (equal faces and edges) to be analyzed in the classroom of basic and high school students

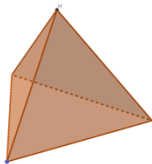
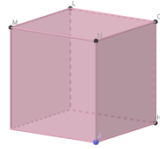
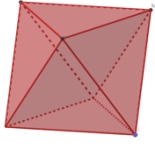
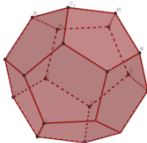
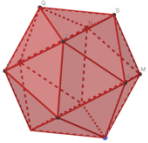
Regular Tetrahedron	Hexahedron Regular (cube)	Hexahedron Regular (cube)	Dodecahedron Regular	Icosahedron Regular
				
four equal equilateral triangles	six equal squares	eight equal equilateral triangles	twelve equal regular pentagons	Twenty equal equilateral triangles

Table 1

Some data to quantify the basic geometric figures, based on the studies of Euler's works

Polyhedron	Face Type	Vertexes	Faces	Edges	Formula
Tetrahedron	Triangle	4	4	6	$4 + 4 - 6 = 2$
Hexahedron Cube	Square	8	6	12	$8 + 6 - 12 = 2$
Octahedron	Triangle	6	8	12	$6 + 8 - 12 = 2$
Dodecahedron	Pentagon	20	12	30	$20 + 12 - 30 = 2$
Icosahedron	Triangle	12	20	30	$12 + 20 - 30 = 2$

2.3. Some Contributions of Carl Friedrich Gauss

Gauss's Sum is a mathematical formula that in a very simple and elegant way determines the sum of the first and consecutive natural numbers; its discovery is attributed to the genius mind of Gauss, who, as the popular legend says, at an early age, dazzled the world with his keen intuition and ability to perceive numerical patterns (Perero, 1994).

The following figure explains how to add the numbers from 1 to 100; the sum of Gauss consists of adding the first number, the one, with the last, the 100, then the second with the second to last, and so on, but with the luck that always gave the same, now Gauss only multiplied and found the result requested by ruthless teacher. Below is an explanatory image and the formula found. Details of the way in which Gauss presents the sum of his authorship, and of great impact on teaching high school students (El blog de mate de Aida, 2024).

The little Gauss came up with the idea of doing the same sum twice, and arranged it like this:

$$1 + 2 + 3 + 4 + \dots + 98 + 99 + 100 = \text{??????}$$

Procedure A:

$$\frac{100}{101} + \frac{99}{101} + \frac{98}{101} + \frac{97}{101} + \dots + \frac{4}{101} + \frac{3}{101} + \frac{2}{101} + \frac{1}{101}$$

$$\sum_1^{100} i/101 \text{ where with } i = 1, 2, 3, 4 \dots 100$$

there are 100 of these sums...

therefore, the total sum is 100 times

101 divided by 2, like this:

$$\text{The total amount} = \frac{100(101)}{2}$$

Procedure B:

$$1 + 100 = 101$$

$$2 + 99 = 101$$

$$3 + 98 = 101$$

.

.

.

$$50 + 51 = 101$$

There is 50 x 101 products or, $(100/2) \times 101$

In general Gauss concludes it:

$$\text{The total amount: } n(n+1) / 2$$

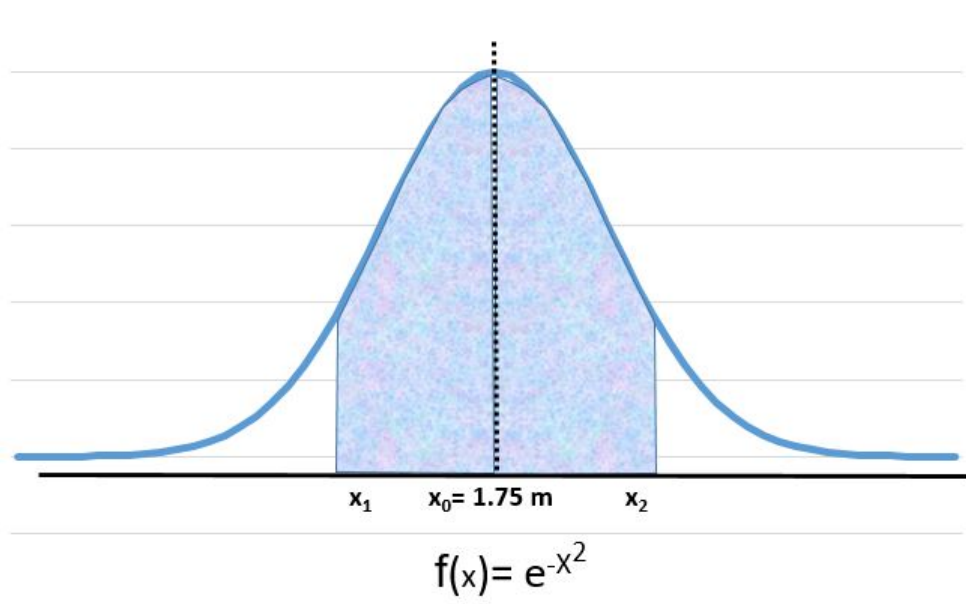
The sum that Gauss figured out...

2.4. The Gaussian Bell

Is a mountain-shaped curve that is a powerful tool in statistics and is related to the averages of large populations. For example (Figure 5): The average age of the Mexican population is 29 years old. This value is in the center, that is, at the top of the curve, then at the extreme right, the curve is very low because there must be very few Mexicans over 90 years old. Another example is the average height of students in a state or country, for example, it may be an average of 1.75 meters and therefore at the right end of the curve, it means that there are few students over two meters tall and the left end has a small population with too short of a height. The peak of the curve represents the most likely or average value. The bell shape indicates that values are more likely near the average, less likely as you approach the extremes of the famous Gaussian curve. The Gaussian Bell (Bühler, 1981) is an essential and effective tool in science today, all disciplines use it, for example, biology and economics.

Figure 5

Form of a bell-shaped geometric model to explain and make statistical decisions based on statistical parameters of centralization and dispersion, of various types of problems



In the context of magnetism, the unit of measurement called *gauss* is used to express the intensity of a force that a magnet has that produces a magnetic field (Dunnington, 2012). One *gauss* is equivalent to 0.0001 *teslas*. The *gauss* is used to describe the strength of a magnet (Figure 6).

Figure 6

Magnetite samples, with magnetic fields, forces and parameters.

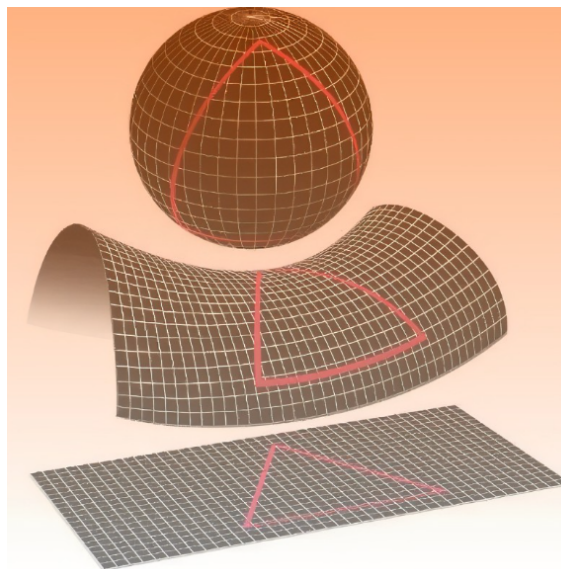


Note. The unit of magnetic field intensity is the gauss in the CGS System

Gauss is also famous for his study of other geometries different from the predominant paradigm of Euclidean spaces. This space means the palpable to the naked eye in 3D space. Nevertheless, there are other types of geometry, such is the case of the so-called hyperbolic geometry, which changes the rules of the normal, the hyperbolic is a curved space, unlike the flat space of Euclidean geometry.

Figure 7

An illustrative form of the Three types of geometry: Elliptical, hyperbolic and planar



3. Conclusions

In this paper, it was proposed a set of topics concerning the biographies of Euler and Gauss that we recommend should be included in the curriculum at the pre-university level. The explanations presented demonstrate that we can facilitate students' understanding. *Euler's Straight Line* may have complications in the concepts involved, but it can be understood in a general way. The formula for regular polyhedral is less difficult to understand. *Gauss's sum* and the *bell* can be explained in a simple way, avoiding complications. Other topics that can be considered are prime numbers, exponential and logarithmic functions in the case of Euler and the method of least squares in Gaussian topics. Finally, the videos by *My Favorite Mathematician and Gauss: The Prince of Mathematics* from the author Eduardo Sáenz de Cabezón are highly recommended.

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